# EE-434 Power Electronics

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# Overview

1. Half-Wave Rectifiers (HWR)
   - Introduction
   - Uncontrolled HWR with Resistive load
   - Uncontrolled HWR- Resistive-Inductive load
2. Freewheeling diode

# Introduction

## What ?

Convert AC-DC

## Why?

Electronic circuits use DC while the generation system is AC

## When?

Half wave rectifiers are seldom used because of low efficiency. However, studying this provide basis for understanding more complicated circuits.

# UCHWR — Resistive load

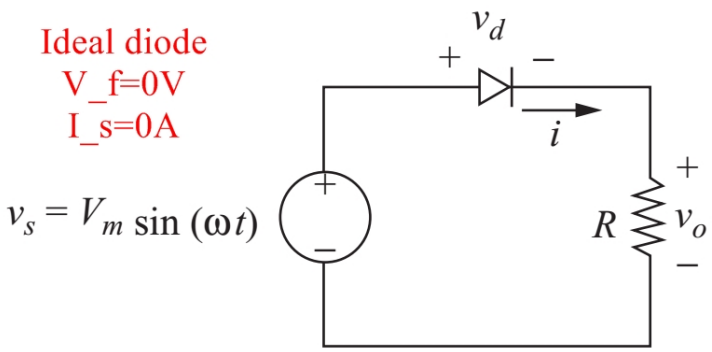



Figure: Circuit diagram

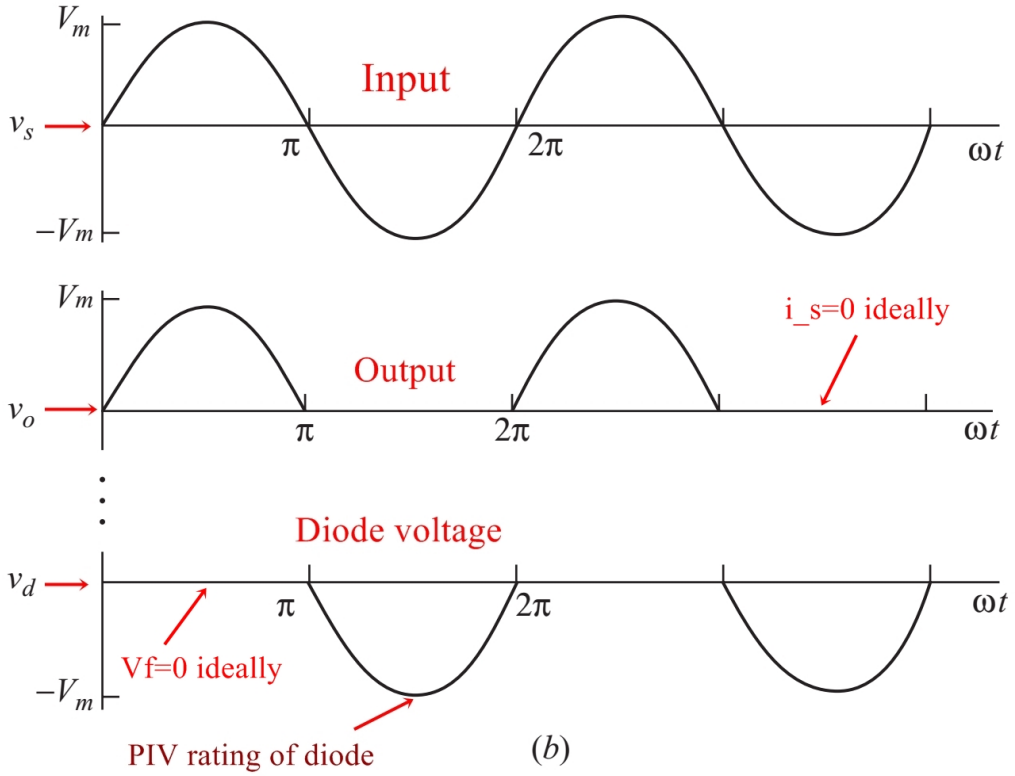



Figure: Key waveforms

$$V_o = V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m sin(\omega t) d(\omega t) = \frac{V_m}{\pi} \tag{1}$$

# UCHWR — Resistive load

Using (1) and Ohm's law, the current is given as

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \tag{2}$$

Using the EE-211 knowledge, the power is computed as

$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R} \tag{3}$$

To find P, the instantaneous voltage $V_o$ and $I_o$ are converted into rms as follows

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_o^{\pi}[V_m sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2} \tag{4}$$

$$I_{rms} = \frac{V_m}{2R} \tag{5}$$

# Example 3-1 UCHWR-R

For the half-wave rectifier, the source is a sinusoid of 120 $V_{rms}$ at 60 Hz. The load resistor is 5Ω. Determine (a)the average load current, (b) the average power absorbed by the load and (c) the power factor of the circuit.

Solution:

(a) The peak voltage across R is $\sqrt{2} \times 120$=169.7V. Average voltage is $\frac{V_m}{\pi}$. The average current is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}120}{5\pi} = 10.8A \quad (6)$$

(b) The rms voltage across the resistor is

$$V_{rms} = 0.707 \times V_m = 84.9V \quad (7)$$

Power absorbed by the resistor is

$$P = \frac{V_{rms}^2}{R} = \frac{84.9^2}{4} = 1440W \quad (8)$$

(c) The power factor is

$$pf = \frac{P}{S} = \frac{P}{V_{s,rms} I_{s,rms}} = \frac{1440}{(120)(17)} \quad (9)$$

pf=0.707

# UCHWR — Resistive-Inductive load

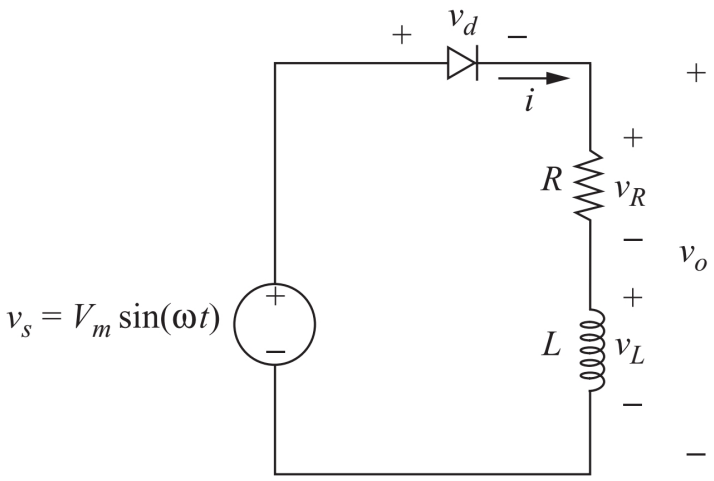
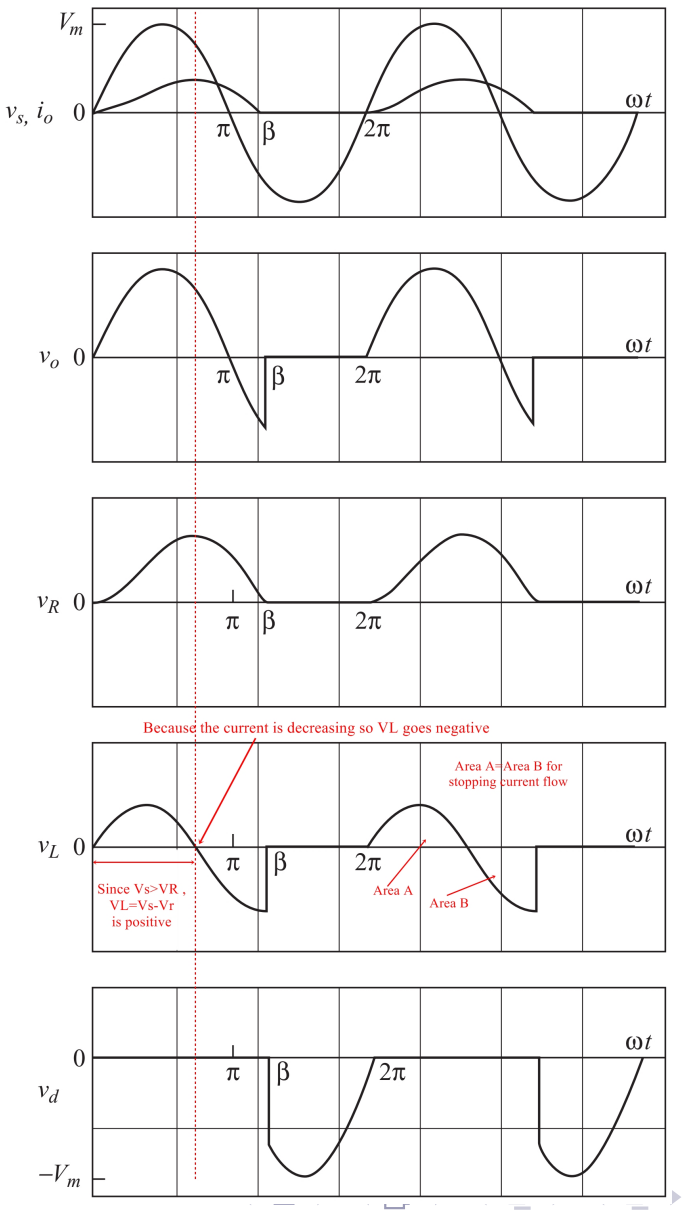



Figure: Circuit diagram

Industrial loads are mostly RL loads. The presence of inductance makes the circuit complex.



# UCHWR — Resistive Inductive load

Applying KVL on the circuit considering ideal diode.

$$V_m sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} \tag{10}$$

This is a first order circuit and therefore, the solution consist of forced and natural response of current.

$$i(t) = steady + transient = forced + naturalresponse \tag{11}$$

The forced response is the steady state response and is given as

$$i_f(t) = \frac{V_m}{Z} sin(\omega t - \theta) \tag{12}$$

where, Z=$\sqrt{R^2 + (\omega L)^2}$ and $\theta = tan^{-1}\left(\frac{\omega L}{R}\right)$.
Natural response comes up without a source or a diode. and therefore the solution is a homogeneous differential equation.

# UCHWR — Resistive Inductive load

$$Ri(t) + L\frac{di(t)}{dt} = 0 \tag{13}$$

This differential equation is solved using .

$$i_n(t) = Ae^{\frac{-t}{\tau}} \tag{14}$$

where, $\tau$ is time constant and is given as L/R. If the value of A is calculated then particular solution can be obtained. This can be evaluated using initial conditions of the circuit.

$$i_(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}sin(\omega t - \theta) + Ae^{\frac{-t}{\tau}} \tag{15}$$

Since, $i_L(0^-) = 0$, therefore,

$$i_(0^+) = \frac{V_m}{Z}sin(0 - \theta) + Ae^0 = 0 \tag{16}$$

# UCHWR — Resistive Inductive load

$$A = \frac{-V_m}{Z} sin(-\theta) = \frac{V_m}{Z} sin\theta \tag{17}$$

This can be used in (15) to get a particular solution as.

$$i(t) = \frac{V_m}{Z} sin(\omega t - \theta) + \frac{V_m}{Z} sin(\theta) e^{\frac{-t}{\tau}} \tag{18}$$

or

$$i(t) = \frac{V_m}{Z} \left[ sin(\omega t - \theta) + sin(\theta) e^{\frac{-t}{\tau}} \right] \tag{19}$$

It is convenient to write functions in terms of $\omega t$. So above equation can be rewritten as

$$i(\omega t) = \frac{V_m}{Z} \left[ sin(\omega t - \theta) + sin(\theta) e^{\frac{-\omega t}{\omega \tau}} \right] \tag{20}$$

This is valid for only positive half cycle because of diode.

# UCHWR — Resistive Inductive load

The important aspect is that diode conducts longer than $\pi$ rad i.e., during a portion of negative half cycle.
However, the circuit follows KVL. It is because the inductor voltage is negative for decreasing current ($V_L = Ldi/dt$).
The current becomes zero when diode turns off. The value of $\omega t$ that results in zero current is called extinction angle $\beta$. So (20) becomes

$$i(\beta) = \frac{V_m}{Z}\left[sin(\beta - \theta) + sin(\theta)e^{\frac{-\beta}{\omega\tau}}\right] = 0 \tag{21}$$

This reduces to

$$sin(\beta - \theta) + sin(\theta)e^{\frac{-\beta}{\omega\tau}} = 0 \tag{22}$$

$$i_{(\omega t)} = \begin{cases} \frac{V_m}{Z}\left[sin(\omega t - \theta) + sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \leq \omega t \leq \beta \\ 0 & \beta \leq \omega t \leq 2\pi \end{cases} \tag{23}$$

# UCHWR — Resistive Inductive load

The average power absorbed by the load is $I_{rms}^2 R$ because average power absorbed by inductor is zero. Negative portion in output voltage reduces the average voltage available.
The rms value of the current is determined from the current function of (23)

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)d(\omega t)} \tag{24}$$

Average current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)d(\omega t) \tag{25}$$

# UCHWR — Resistive-Inductive load simulation

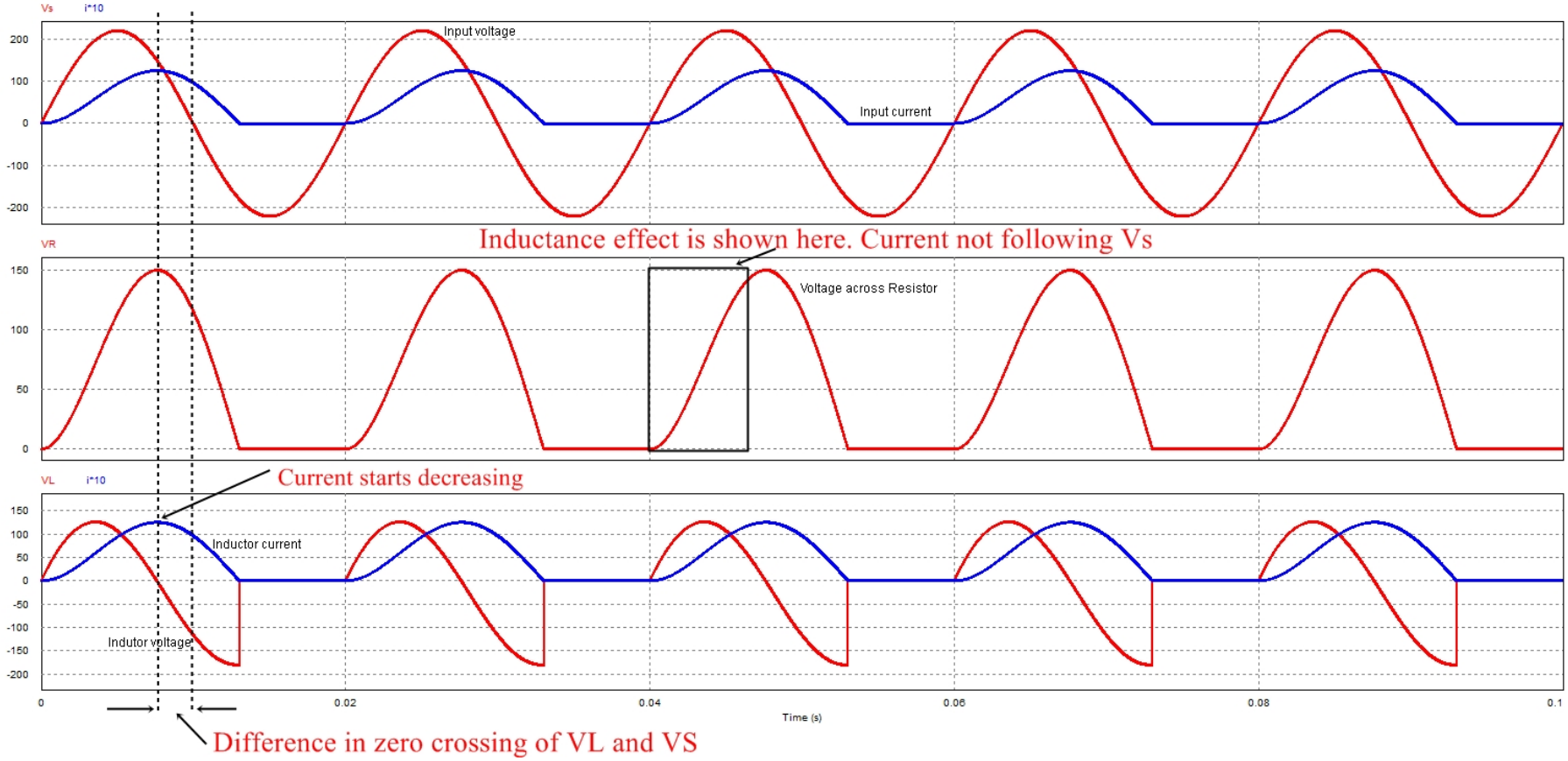



Figure: Simulation results of a HWR in PSIM

# UCHWR — Resistive-Inductive load simulation

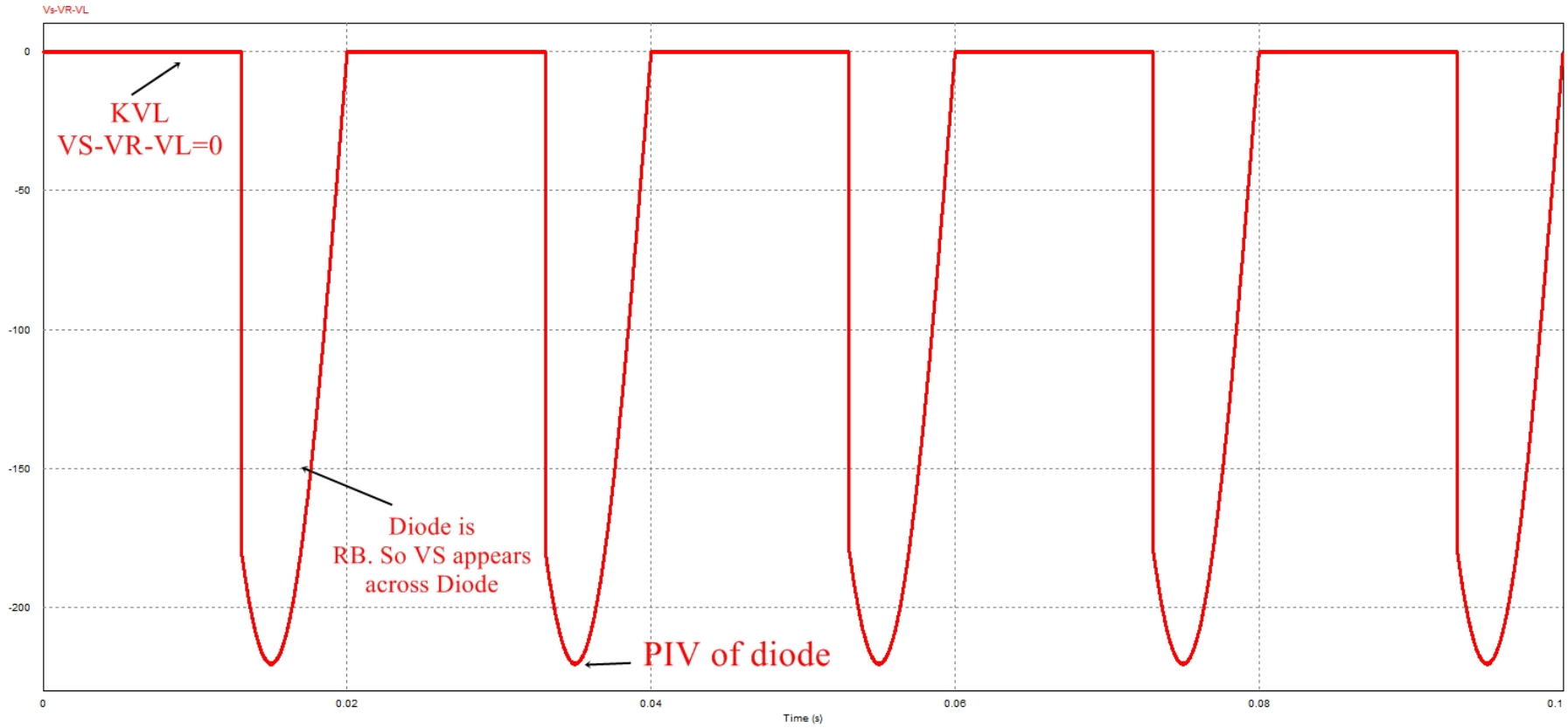



Figure: Simulation results of a HWR in PSIM

KVL is satisfied for the conduction duration. When the conduction stops, all the input voltage is blocked by diode and can be seen here.

# UCHWR — Resistive-Inductive load simulation

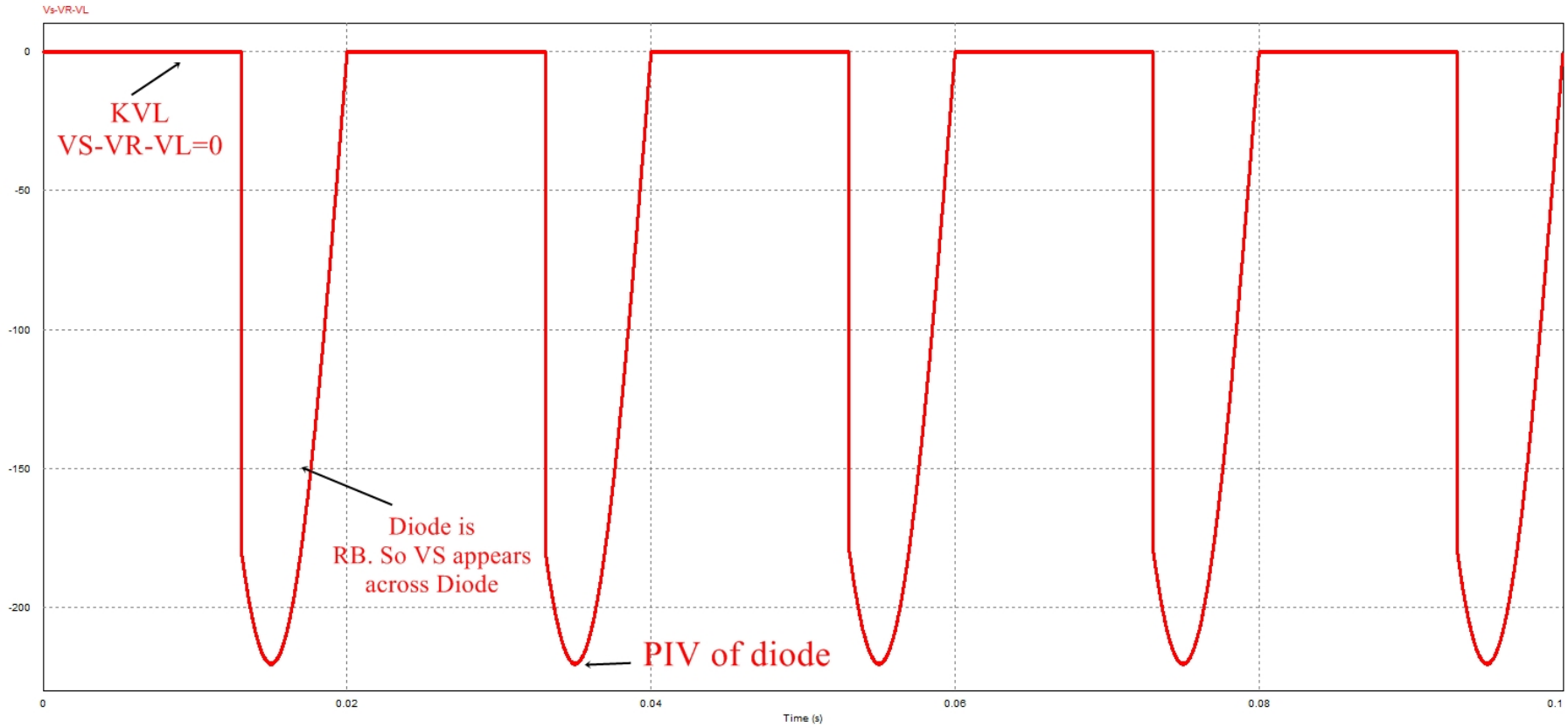



Figure: Simulation results of a HWR in PSIM

KVL is satisfied for the conduction duration. When the conduction stops, all the input voltage is blocked by diode and can be seen here.

# UCHWR — Resistive-Inductive load simulation

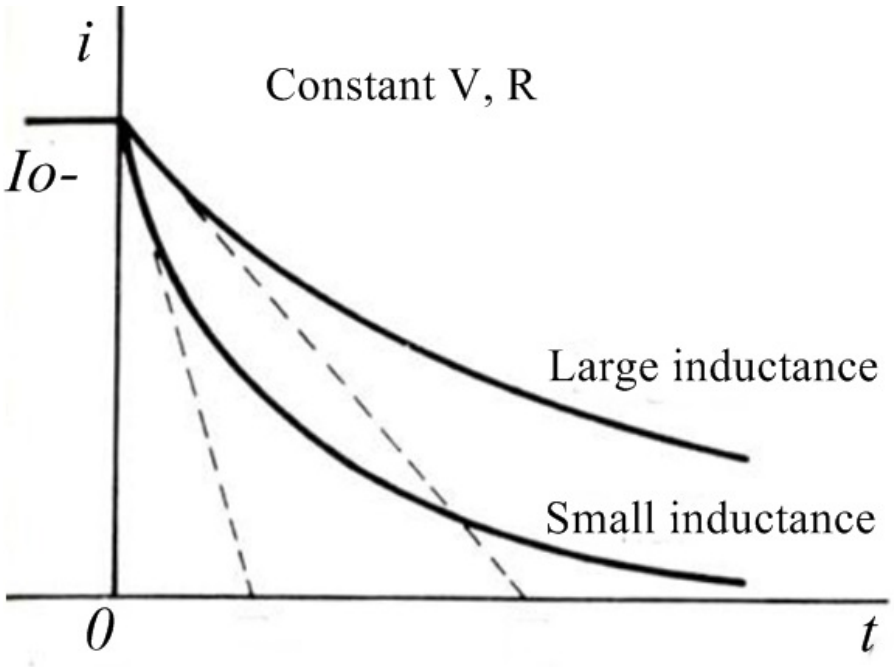



Figure: Effect of inductance

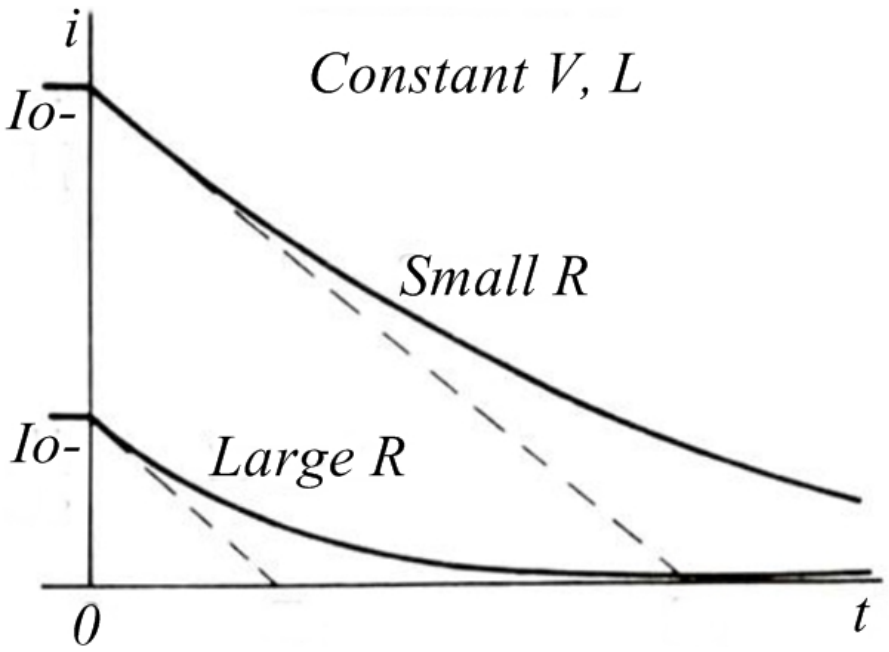



Figure: Effect of resistance

Ref: Network Analysis by Van Valkenburg

# Example 3-2 UCHWR-RL

For the half-wave rectifier of R=100Ω, L=0.1H, $\omega$=377 rad/s, and $V_m$=100 V. Determine (a) current expression (b) the average current, (c) the rms current, (d) the power absorbed by the RL load, and (e) the power factor.

Solution:

For this circuit following parameters are calculated

$$Z = [R^2 + (\omega L)^2]^{0.5} = 106.9 \tag{26}$$

$$\theta = tan^{-1}(\omega L/R) = 20.7^\circ = .361 rad \tag{27}$$

$$\omega t = (\omega L/R) = .377 rad \tag{28}$$

Using (23) the current expression can be obtained

$$i(\omega t) = 0.936 sin(\omega t - 0.361) + 0.331 e^{-\omega t/0.377} = 0 \tag{29}$$

Calculation of beta require some root-finding program and is equal to 3.5rad or 201°.

# Example 3-2 UCHWR-RL

Average current is determined from (25)

$$I_o = \frac{1}{2\pi}\int_0^{3.5}[0.936sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}]d(\omega t) = 0.308A \tag{30}$$

The rms current is calculated from (24)

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{3.5}[0.936sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}]^2 d(\omega t)} = 0.474A \tag{31}$$

The power absorbed by the resistor is

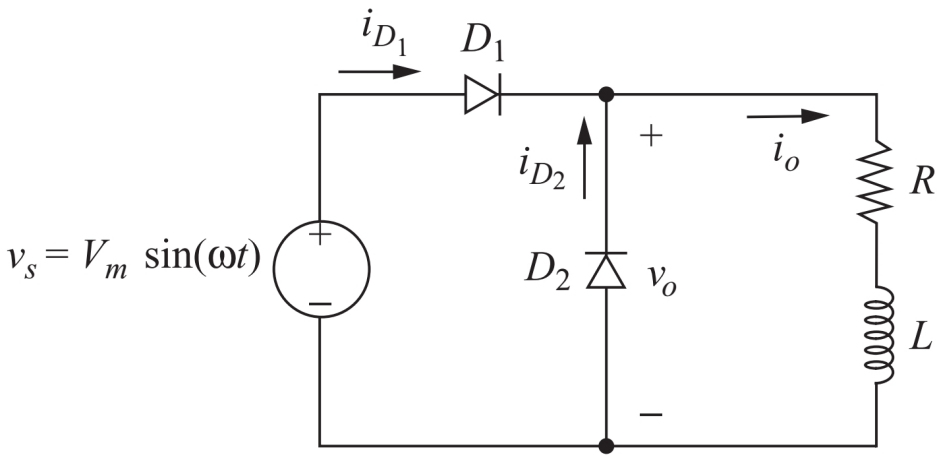
$$P = I_{rms}^2 R = (0.474)^2(100) = 22.4W \tag{32}$$

Note that average power absorbed by the inductor is zero.
Power factor is given by

$$pf = P/S = \frac{P}{V_{s,rms}I_{s,rms}} = \frac{22.4}{(100/\sqrt{2})0.474} = 0.67 \tag{33}$$

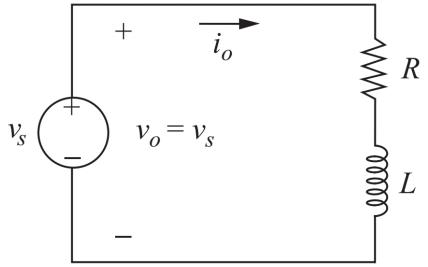
# Freewheeling diode

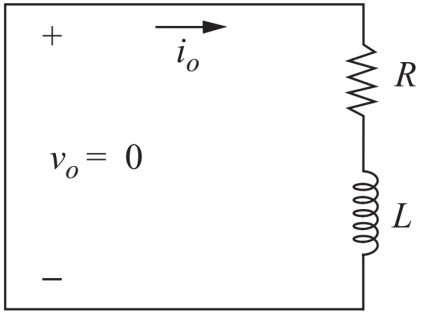
For the inductive load, the presence of negative voltage cause the average output to decrease.
Moreover, the inductive voltage create stress on the power electronic switches.




When $D_1$ conducts, $D_2$ is reverse biased. When $D_1$ is reverse biased, the $D_2$ conducts.

# Freewheeling diode




For $0 \leq \omega t \leq \pi$ $D_1$ is on and $D_2$ is off. The circuit is of the form as shown.
The voltage across the load is same as the source.

For $\pi \leq \omega t \leq 2\pi$ $D_1$ is off and $D_1$ is on. The circuit is of the form as shown.
The voltage across the RL load is zero

The Fourier series for half-wave rectified sine wave for output voltage is

$$v(t) = \underbrace{\frac{V_m}{\pi}}_{\text{DC part}} + \underbrace{\frac{V_m}{2} sin(\omega_o t)}_{\text{Fundamental}} - \underbrace{\sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi} cos(n\omega_o t)}_{\text{Even harmonics}} \tag{34}$$

# Freewheeling diode

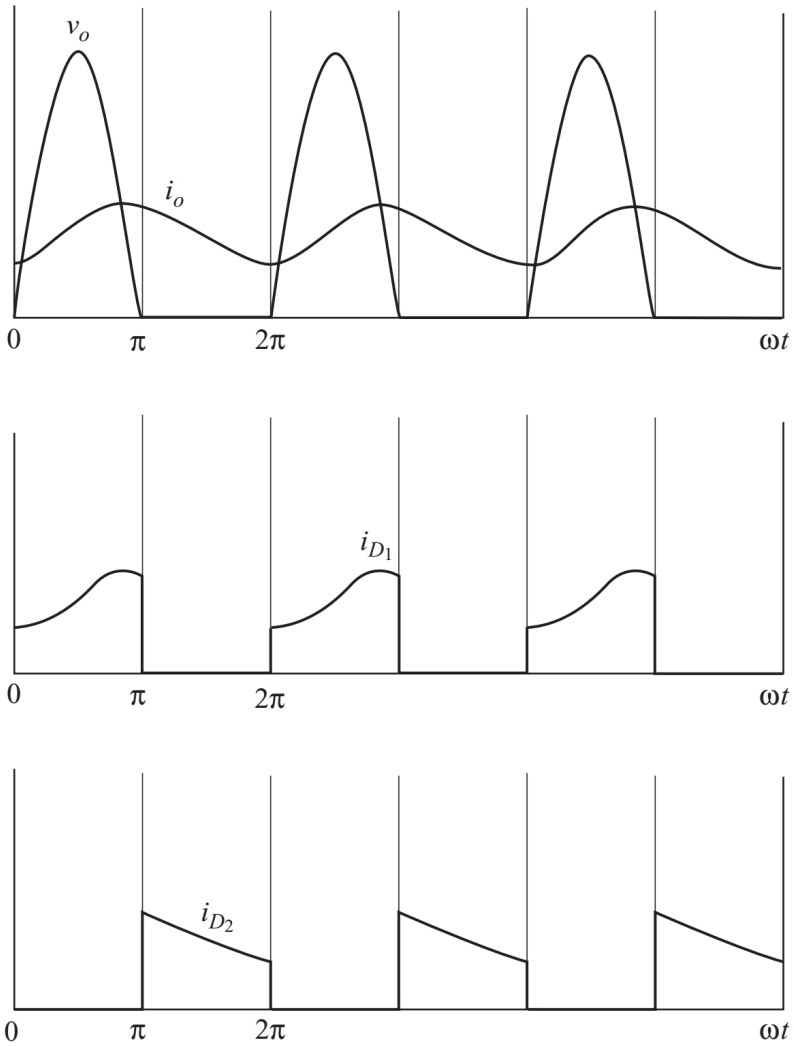



Figure: Key waveforms

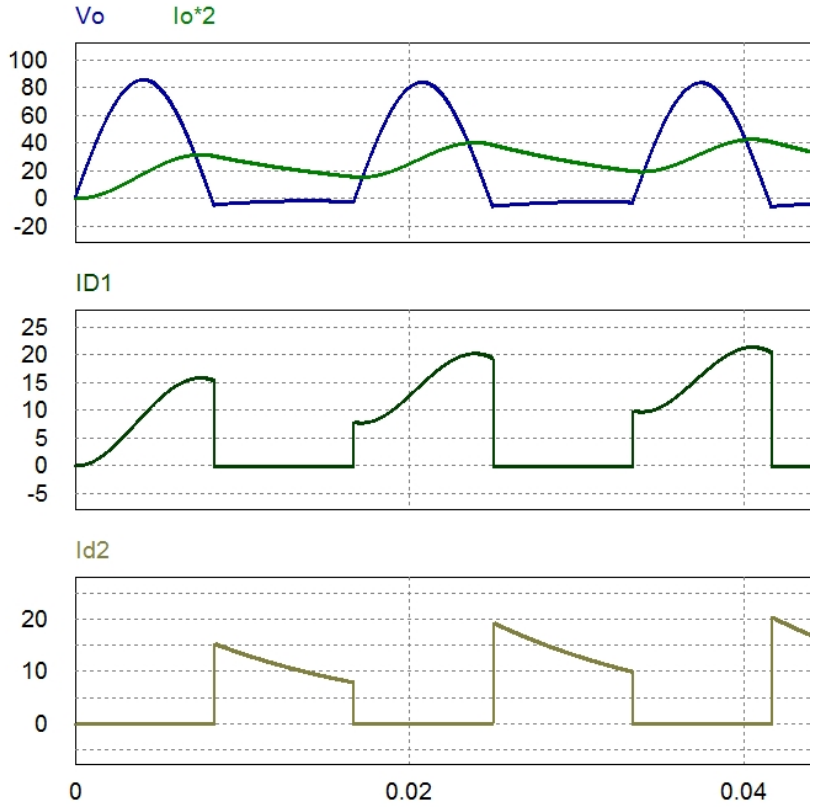



Figure: Simulated waveforms R=2, L=25m, Vp=100 f=60

# Example 3-7 — Freewheeling diode

For a half wave free wheeled rectifier, determine the average load voltage and current, the power absorbed by the resistor. Note that R=2Ω and L=25 mH, $V_m$=100 V, and the frequency is 60 Hz.

Solution

The average voltage is the dc term of (34)

$$V_o = \frac{V_m}{\pi} = 31.8V \qquad (35)$$

The average load current is

$$I_o = \frac{V_o}{R} = 15.9A \qquad (36)$$

Load power is $I_{rms}^2 R$. The rms current is determined from the Fourier components of current. AC current components are determined from phasor analysis

$$I_n = \frac{V_n}{Z_n} \qquad (37)$$

where, $Z_n = |R + jn\omega_o L| = |2 + jn377(0.025)|$.

AC voltage resulting from (34) are

$V_1 = \frac{V_m}{2} = 50V$

$V_2 = \frac{2V_m}{(2^2-1)\pi} = 21.2V$

$V_4 = \frac{2V_m}{(4^2-1)\pi} = 4.24V$

$V_6 = \frac{2V_m}{(6^2-1)\pi} = 1.82V$

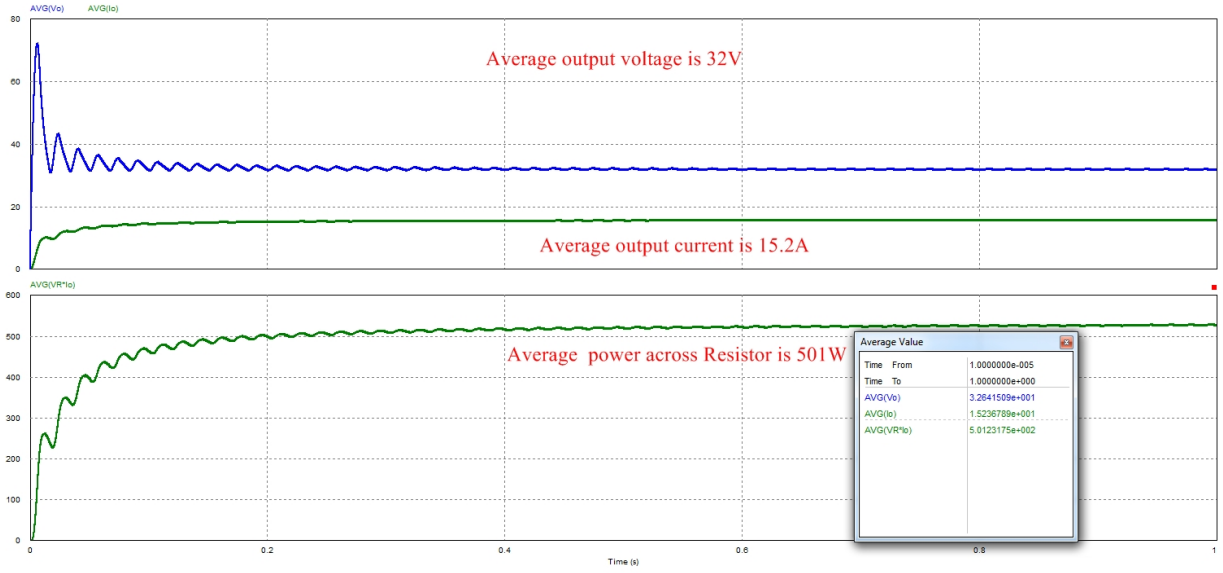
# Freewheeling diode

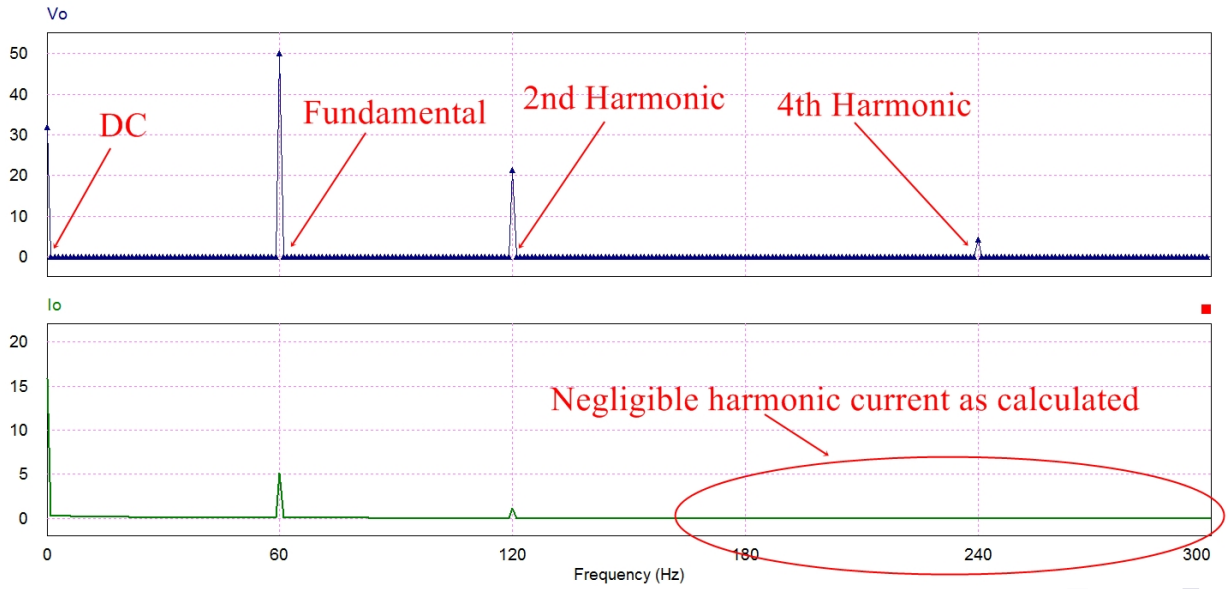
So, the respective current value from fourier terms are $I_0 = \frac{V_0}{Z_0} = 15.9A$
$I_1 = \frac{V_1}{Z_1} = 5.19A$
$I_2 = \frac{V_2}{Z_2} = 1.12A$
$I_4 = \frac{V_4}{Z_4} = 0.11A$
$I_6 = \frac{V_6}{Z_6} = 0.03A$
The rms current is obtained as

$$I_{rms} = \sqrt{\sum_{k=0}^{\infty} I_{k,rms}} \approx \sqrt{15.9^2 + \left(\frac{5.19}{\sqrt{2}}\right)^2 + \left(\frac{1.12}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2} = 16.34A \tag{38}$$

Power absorbed by the load is $I_{rms}^2 R = 534W$

# Example 3-7 —Simulation



# Reducing load current harmonics

The output current is given as

$$I_{output} = I_{dc} + I_{ac} \tag{39}$$

The dc component is desired whereas, ac component constitutes the ripples.The average load current (dc) in the RL load depends only on **RESISTANCE**, owing to the **Ohm's law**.This means that the inductance has no impact on the average (dc) current.However, because of inductive reactance ($X_L = 2\pi fL$), the inductance do have an impact on the ac components. If the inductance is made large enough, the impedance to the ac components will become high, consequently, the output become a pure dc.

$$i_o(t) \approx I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \qquad \frac{L}{R} \implies \infty \tag{40}$$

A large inductor means ($\tau >> T$) means a constant load current.

$$\Delta I_o \approx 2I_1 \tag{41}$$

# Example 3-8 — Freewheeling diode with $\tau = \infty$

For a half wave free wheeled rectifier, determine the average current in each diode, the power absorbed by the resistor and power factor. Note that R=8Ω and L=25H(assumed very large), $V_{rms}$=240 V, and the frequency is 60 Hz. Later, find L such that the $\Delta I_{p-p}$ is 10 percent of the average current.

Solution

(a)The average current is given in (40)

$$i_{\omega t} = I_o = \frac{V_m}{\pi R} = 13.5A \quad (42)$$

This is also the rms current.

The power in resistor is $I_{rms}^2 R$=1459W. The source rms current is

$$I_{s,rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(13.5)^2 d(\omega t)} = 9.55A \quad (43)$$

Power factor is

$$pf = \frac{P}{V_{s,rms} I_{s,rms}} = 0.637 \quad (44)$$

Each diode conducts for one half cycle. So average current for each diode is $I_o/2$=6.75A

# Freewheeling diode

So, the respective current value from fourier terms are $I_0 = \frac{V_0}{Z_0} = 15.9A$
$I_1 = \frac{V_1}{Z_1} = 5.19A$
$I_2 = \frac{V_2}{Z_2} = 1.12A$
$I_4 = \frac{V_4}{Z_4} = 0.11A$
$I_6 = \frac{V_6}{Z_6} = 0.03A$
The rms current is obtained as

$$I_{rms} = \sqrt{\sum_{k=0}^{\infty} I_{k,rms}} \approx \sqrt{15.9^2 + \left(\frac{5.19}{\sqrt{2}}\right)^2 + \left(\frac{1.12}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2} = 16.34A \tag{45}$$

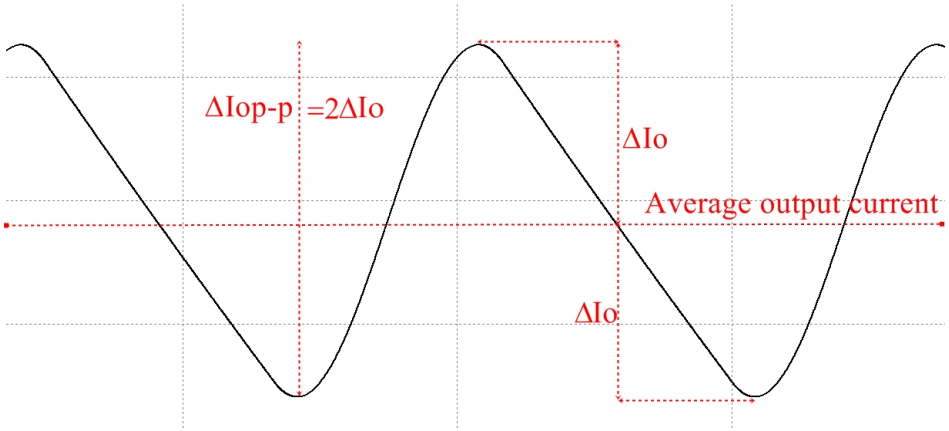
Power absorbed by the load is $I_{rms}^2 R = 534W$

# Example 3-8 — Freewheeling diode with $\tau = \infty$

The value of inductance required to ensure the ripple current of 10 percent can be calculated using fourier series.
The voltage input to load of n=1 has amplitude of
$V_m/2 = \sqrt{2}(240)/2 = 170V$.
Also, the peak to peak current must be limited to

$$\Delta I_o = (0.10)(I_o) = (0.10)(13.5) = 1.35A \tag{46}$$

This corresponds to peak ripple of 13.35/2=0.675A



# Example 3-8 — Freewheeling diode with $\tau = \infty$

The load impedance at fundamental frequency is

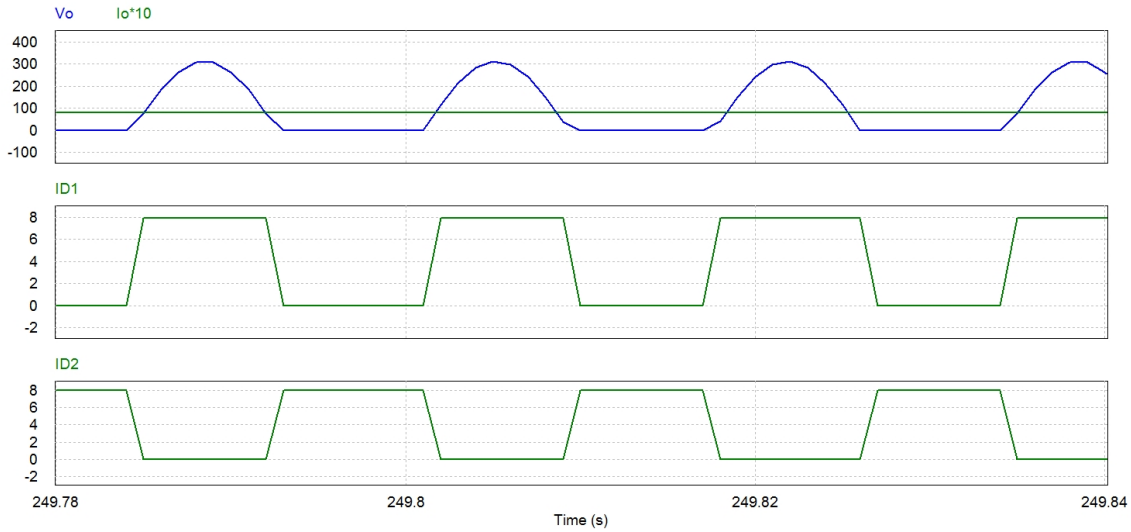
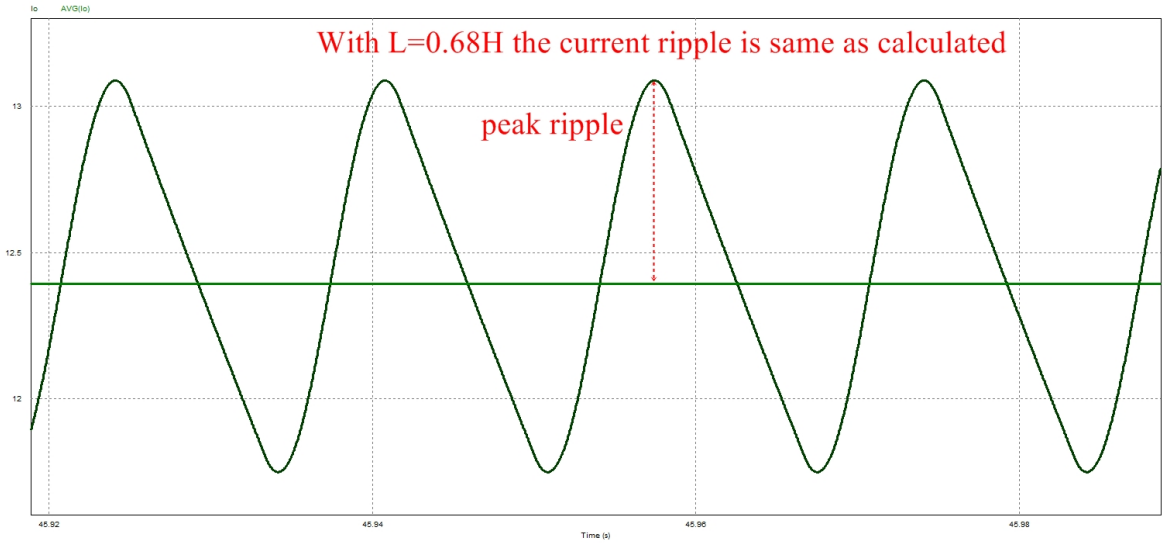
$$Z_1 = \frac{V_1}{I_1} = \frac{170}{0.675} = 251\Omega \tag{47}$$

The load impedance is

$$Z_1 = 251 = |R + j\omega L| = |8 + j377L| \tag{48}$$

As resistance is very less, therefore,

$$L \approx \frac{Z_1}{\omega} = \frac{251}{377} = 0.67H \tag{49}$$

To compensate the impact of harmonics, L is chosen slightly higher than this value e.g., 0.68H

# Example 3-8 —Simulation